Long Oblique Internal Hydraulic Jumps at the Penghu Channel of the South China Sea

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Motivation of this study

- To explain some internal wave-like features identified in SAR images at the Penghu Channel of the South China Sea, a shallow canal with a lateral constriction in between Taiwan and the Penghu Islands.
- Investigate the three-dimensionality of the phenomenon
- Investigate its frequency of occurrence and relevance to physical oceanography
- Demonstrate that SAR satellite remote sensing may offer privileged means of observation and retrieval of physical oceanography measurements
Introduction: hydraulic jumps

A tidal bore (in a river) is a wall of water about a meter high travelling rapidly upstream as the tide floods in.

It occurs where the tidal range is large and the estuary is funnel-shaped.

Hydraulic jumps can be seen in the kitchen sink. When the water from the tap hits the sink pan, it flows radially and rapidly outwards in a thin layer until it slows to a critical speed where a circular hydraulic jump, the sink bore, forms.

The critical speed is determined by a quantity called the Froude number. The Froude number Fr is a ratio of speeds,

\[ Fr = \frac{\text{Characteristic flow speed}}{\text{Gravity wave speed}} \]

A hydraulic jump occurs where the Froude number equals 1.

A supersonic aircraft generates a cone-shaped disturbance with apex angle \( 2\theta \) such that \( \sin \theta = 1/M \) where M is the Mach number, the ratio of aircraft speed to the speed of sound.

A similar triangular region can easily be formed in the sink: put a needle into the inner disk of water and a V-shaped disturbance forms.

Hypothesis: Mach waves and oblique internal shock waves

\[ M = \frac{U}{c} = \frac{1}{\sin \mu} \]

\( M \) is the Mach number and \( \mu \) is the Mach angle.
Introduction: methodology
For internal waves in a two layer system the propagation speed is

\[ c_{\pm} = \frac{u_1 d_2 + u_2 d_1}{d_1 + d_2} \pm \left( \frac{g' d_1 d_2}{d_1 + d_2} \right)^{1/2} \left( 1 - \frac{(u_2 - u_1)^2}{g'(d_1 + d_2)} \right)^{1/2} = \bar{u} \pm \tilde{c}_0 \quad (1) \]

- where \( d_i, u_i \) and \( \rho_i \) are the depth, speed, and density of the upper \((i=1)\) and lower layers \((i=2)\). \( g' = g(\rho_2 - \rho_1)/\rho_2 \) is the reduced gravity

**CRITICAL CONDITION:** \( c_0 = 0 \)

**FOR UPSTREAM-MOVING INTERNAL WAVES**

\[ F = \frac{\bar{u}}{\tilde{c}_0} = 1 \]
$B = \frac{-g(z - \bar{z})}{\rho_0}$

Meridional component

Vertival eigenfunction

CRITICAL CONDITION: $c_- = 0$
Case Study №1: 1999.07.15; ERS-SAR

$\mu_{\text{obs}} = 48.5^\circ$

Upstream features
Case Study nº1: 1999.07.15; ERS-SAR

\[ \mu_{\text{pred}} = \sin^{-1}(1/F_r) = 42^\circ \]

\[ \mu_{\text{obs}} = 48.5^\circ \]
Case Study Nº1: 1999.07.15; ERS-SAR

Downstream features
Standard BVP is solved for the downstream stratification, in its non-hydrostatic limit ($\lambda=200\text{m}$), according to the Taylor-Goldstein equation (below):

$$\frac{d^2 \phi}{dz^2} + \left[ \frac{N^2(z)}{(U - c)^2} - \frac{d^2 U / dz^2}{U - c} - k^2 \right] \phi = 0, \quad \phi(0) = \phi(-H) = 0$$

- $\Phi$ is the modal structure function, $k$ is the wavenumber (also called the non-hydrostatic term), $U$ is the velocity of the flow along the waves’ direction of propagation (i.e. barotropic tides + steady currents = 0.8 m/s), and $H=120\text{m}$ is the local depth, which must be considered constant.
- In the non-hydrostatic approximation ($k \neq 0$), assuming $\lambda = 200\text{ m}$ obtained from the SAR images, the phase speed of the first mode is: 0.5 m/s.
Case Study Nº1: 1999.07.15; ERS-SAR

$\mu_{\text{pred}} = \sin^{-1} \left( \frac{1}{Fr} \right) = 16^\circ$

$\mu_{\text{obs}} = 18^\circ$
\[ \mu_{\text{obs}} = 52.5^\circ \]

\[ \langle Fr \rangle = \sin^{-1}(1/\mu_{\text{obs}}) = 1.26 \]

\[ c_{\text{pred}} = \frac{u_{\text{tide}} + u_{\text{steady}}}{Fr} \]

\[ c_{\text{pred}} \approx 0.33 \text{ m/s} \]

(October stratification)

Case Study Nº2: 2017.10.08; Sentinel-1A (IW GRD)
Case Study Nº3: 2009.08.14; ENVISAT (IMP) ; **NO SHOW**

Fr < 1 ; Flow is subcritical
No waves are observed
Tidal flow is to the South, but total mean flow is still Northward!
Synergy possibilities
Supersonic flow compressed by a concave surface: the free-stream Mach number is 1.96. From station 1 to station 2 the flow is turned 20 degrees.

\[
\frac{c_2}{c_1} = \frac{\sin(\mu_2)}{\sin(\mu_1)}
\]

\[
\frac{c_2}{c_1} = 1.48
\]

rather large for being an effect of maximum-amplitude bore speed,

\[
c_2/c_1 \approx 1.1
\]
Conclusions

• Internal wave-like features observed in the Penghu Channel are consistent with Mach wave theory, analogous to oblique shock waves in a stratified ocean environment.

• The flow is supercritical most of the times, due to a strong steady current superposed to equally strong tidal currents (with important semi-diurnal and diurnal constituents).

• When the total (steady + tidal) flow relaxes to subcritical conditions, no internal wave activity is observed, at times.

• Analogy with shock wave theory proved insightful to interpret the observed phenomenon in the SAR.

• SAR observations of Mach angles may be used to infer the vertical density stratification in the Channel.
Thank you!